

# *Influence of parameters of the flanged open-ended coaxial probe measurement setup on permittivity measurement*

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**Abstract:** The flanged open-ended coaxial probe is studied using a full-wave model. Influence of parameters like a gap, sample thickness, set-up measurement geometry, probe impedance, size of the flange and size of the sample are investigated and presented. Study is limited to dielectric materials with different characteristics (low loss, high loss). The results showed that error in an air gap is the most important parameter that affects the permittivity measurement accuracy, but also several other parameters are important and present considerable constraints regarding application of open-ended coaxial probe. We also identified the optimal measurement geometry in order to minimize the effect of these parameters.

**Key words:** full-wave model, dielectric materials, open-ended coaxial probe

## *Vpliv parametrov merilnega sistema z odprto koaksialno sondo na meritve dielektričnosti*

**Povzetek:** Predstavljena je študija odprte koaksialne sonde s prirobnico z uporabo modela polnovalne analize. Raziskali in predstavili smo vpliv parametrov kot so reža, debelina vzorca, merilna geometrija, impedanca sonde, velikost prirobnice in velikost vzorca. Študija je omejena na dielektrične materiale z različnimi karakteristikami (nizko izgubne, visoko izgubne). Rezultati so pokazali, da ima največji vpliv na merilno točnost meritve dielektričnosti zračna reža, poleg tega pa so pomembni tudi ostali parametri, ki predstavljajo precejšnje omejitve aplikacij z odprto koaksialno sondo. Prav tako smo poiskali optimalno merilno geometrijo, da bi zmanjšali efekt parametrov merilnega sistema z odprto koaksialno sondo.

**Ključne besede:** polnovalni model, dielektrični materiali, odprta koaksialna sonda

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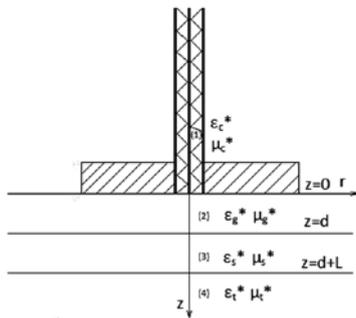
### *1. Introduction*

Each material has distinct dielectric properties and knowing these properties enables engineers to use appropriate materials in specific application. Measuring and understanding how dielectric properties of material vary at microwave frequencies is important in many fields like wireless communication, radar detection or biomedical application.

Intensive studies have been done in development of measurement of the complex permittivity. Many factors like frequency range, required measurement accuracy, sample size, surface topology, state of the material (liquid, solid, powder, thin film), destructive or nondestructive nature of measurements, have to be considered when choosing appropriate method for measurement permittivity. Methods

commonly used include transmission/reflection and resonance methods. Transmission/reflection methods have advantage over resonance methods because they have wide frequency range, are simple to use and can measure lossy materials but are less accurate than resonance methods /1-4/. On the other hand resonance methods are limited to discrete frequencies, defined by resonator dimensions, and to materials with low losses. For transmission/reflection method there are several different approaches by using coaxial waveguide /5, 6/, planar waveguide /7, 8/, rectangular waveguide /9-10/ or free-space method /11-13/. The latter two present frequency limitations due to size of the tested sample and planar waveguides methods are limited to the solid and thick film materials. Hence the most widely used method among transmission/reflection methods is open-ended coaxial line due to its simplicity and accuracy in broadband measurements.

An alternative characterization method based on the reflection is application of an open-end coaxial probe /14/. Simple set-up and sample geometry present significant advantage over other methods; however, for determination of the sample material parameters a good model of wave propagation is needed. There are several models for open-ended coaxial probe like capacitance model /5, 15/, radiation model /16/, virtual line model /17/, rational function model /18, 19/ and full-wave model /14, 20, 21, 24/, with increasing accuracy and also complexity.



**Figure 1:** Flanged open-ended coaxial probe measurement setup with layered dielectric sample with termination.

The aim of this paper is to analyze in detail the open-end coaxial probe system and determine which parameters are affecting measured reflection coefficient for given measurement geometry and evaluate the effect of individual parameter. The analysis is made with the full-wave model and thus presents the most accurate analytical representation of the open-end coaxial probe. We focused the study to non magnetic materials as they are both more commonly measured and require simpler measurement set-up compared to the magnetic materials. In addition, we analyzed the system for both liquid and solid samples and for materials that have either low or high dielectric losses. Our results show the effect of different parameters on permittivity (reflection coefficient) and therefore help researchers to select appropriate measurement geometry for measurement solid materials (low and high loss).

## 2. Methods

We focused on the non-magnetic materials where the tested sample has permittivity  $\epsilon_s^* = \epsilon_0[\epsilon_s' - j\epsilon_s'']$  and permeability  $\mu_s^* = \mu_0$ ,  $\epsilon_0$  is permittivity of vacuum and  $\mu_0$  is permeability of vacuum. The geometry of the problem as shown in Fig.1 consists of an internal and external region. The internal region represents interior of an open-ended coaxial line, while external region is a layered medium. The coaxial line has inner diameter  $2a$ , outer diameter  $2b$  and is filled with a low loss material of permittivity  $\epsilon_c^* = \epsilon_0\epsilon_c'$  and permeability  $\mu_c^* = \mu_0$ . To determine the influence of parameters on measured permittivity we

calculate reflection coefficient with full-wave model /14/ and then use this value in optimization algorithm (Matlab's fsolve trust-region-dogleg algorithm) as a substitute for a measured reflection coefficient. To eliminate measurement uncertainties and have well defined geometry we decided to use model instead of actual measurements. Each parameter of interest was varied to get results (permittivity) and afterwards we compared true value which was used to calculate the true reflection coefficient with the one obtained from optimization.

$$\Delta\epsilon = \left| 1 - \frac{\epsilon_1}{\epsilon_r} \right|, \quad (1)$$

$$\Delta d = d_r - d_i, \quad \Delta L = L_r - L_i, \quad (2)$$

where  $\epsilon_r$  is true value of permittivity,  $\epsilon_1$  is the value obtained with optimization,  $d_r$  and  $d_i$  are values of true gap thickness and gap thickness used in optimization,  $L_r$  and  $L_i$  are values of true sample thickness and sample thickness used in optimization algorithm. The true permittivity of each material was calculated for two probes with following parameters of  $a = 1.51$  mm,  $b = 4.90$  mm,  $\epsilon_c = 1.99$  (realistic 50  $\Omega$  coaxial probe 1),  $a = 0.255$  mm,  $b = 0.84$  mm,  $\epsilon_c = 2.04$  (realistic 50  $\Omega$  coaxial probe 2),  $\epsilon_t = 1$ ,  $d = 50$   $\mu\text{m}$ ,  $L = 500$   $\mu\text{m}$  and 12  $\text{TM}_{0n}$  modes were used. In our study we used values for different materials such as Teflon ( $\epsilon_r = 2-0.003j$  at 10 GHz), mixture of titanium dioxide and wax ( $\epsilon_r = 10-0.09j$  at 10 GHz) and mixture of graphite and wax ( $\epsilon_r = 40-25j$  at 10 GHz) to show the effect of low and high loss material on the relative error in permittivity.

We also used different measurement set-up geometries to find out which would be optimum for dielectric materials. The full-wave model has three distinct measurement set-up geometries, semi-infinite (model 0), short-circuit (model 1) and dielectric terminated (model 2) geometry.

## 3. Results and discussion

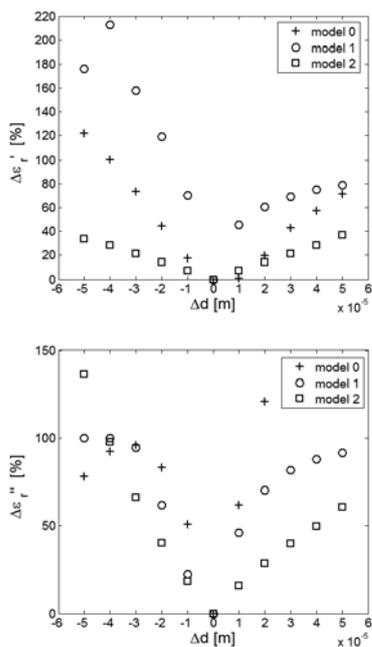
### Influence of parameters on permittivity

The full-wave model used for calculation of reflection coefficient as mentioned earlier is exact but in the case of real measurement set-up it has some disadvantages due to the assumption that the flange and sample extend to infinity in radial direction. These conditions are never satisfied in real measurement, however, the effect of using finite sample and flange in radial dimensions was investigated by De Langhe et al. /22/. It was found that if the ratio between the aperture size and the surface of the sample is greater than 2.5, the measured characteristics (amplitude and phase) are very close to those of the infinite sample. If the sample thickness decreases, one sees larger differences and if thickness increases, the differences get smaller. It was also found

that if the flange radius is at least two times larger than the outer radii of the coaxial probe, only small differences are seen in amplitude and phase. Thus accurate measurements can be made with reasonable flange and sample dimensions despite assumption of infinity.

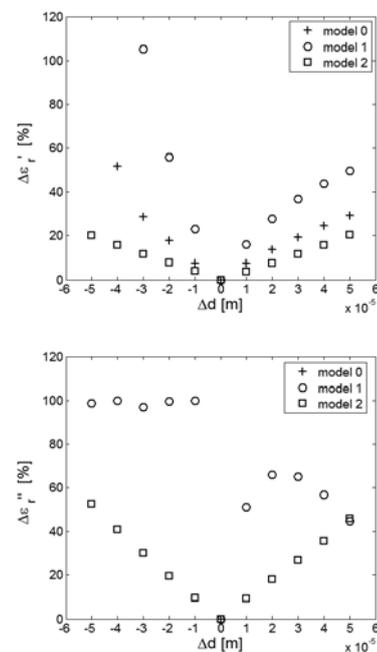
*Influence of gap and sample thickness with different measurement set-up geometry*

One of the important uncertainties is a gap between the sample and the probe as this gap is very difficult to measure. But in reality sample can be also concave or convex and this effect was investigated by A.-K. A. Hassan et al. /23/. It was found that for the concave sample the reflection coefficient is strongly affected by both flange diameter and the radius of concave sample, whereas in the case of convex sample reflection coefficient is affected for small radii of the sample, but the flange diameter has negligible effect on the reflection coefficient. It is concluded that an improve technique is required to achieve better accuracy of measurement of concave samples, while measurements of convex samples, in general, are in good agreement with published data. In order to evaluate how variation of an air gap and sample thickness affect complex permittivity a number of calculations with different measurement set-up geometries were made. In Fig. 2 we compare how error in air gap influences real and imaginary part of permittivity of high loss sample (graphite in wax composite) at 10 GHz.

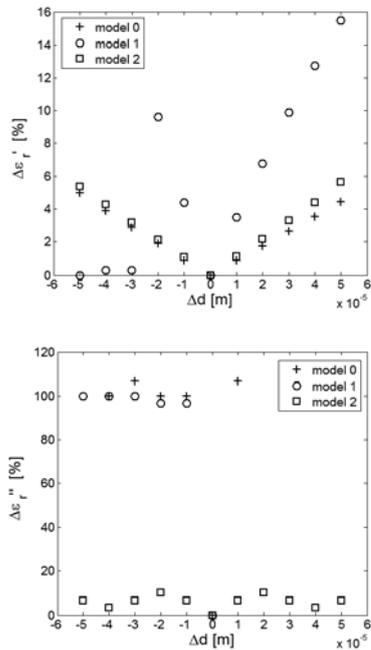


**Figure 2:** Error in real and component of permittivity in percentage as a function of an air gap for a frequency of 10 GHz for graphite mixed with wax with different measurement set-up geometries.

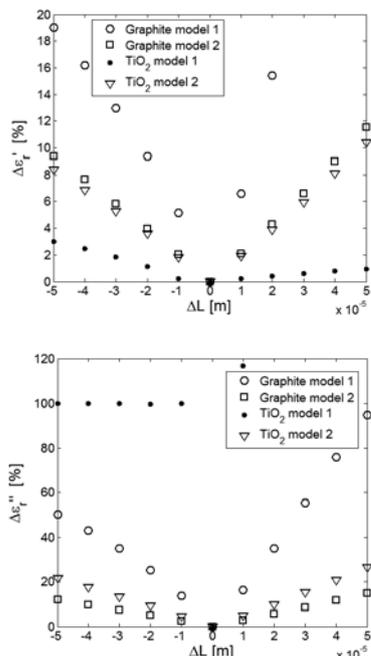
For graphite composite model 2 has the lowest dependence of an air gap on permittivity, nevertheless, error is for both components of permittivity over 40% at air gap value  $\Delta d = 50\mu\text{m}$ . One can see similar results with model 2 for composite of titanium dioxide and wax (Fig. 3). Fig. 4 shows that model 0 and model 2 have similar error in real component of permittivity of teflon, while error in imaginary component clearly shows that model 2 produces better results when the air gap is varied. From Figs. 2-4 one can conclude both that model 2 has in general the least amount of relative error, and that the permittivity of high-loss material is more affected by the air gap.



**Figure 3:** Error in real and imaginary component of permittivity in percentage as a function of an air gap for a frequency of 10GHz for titanium dioxide mixed with wax with different measurement set-up geometries.



**Figure 4:** Error in real and imaginary component of permittivity in percentage as a function of an air gap for a frequency of 10GHz for teflon with different measurement set-up geometries.



**Figure 5:** Error in real and imaginary component of permittivity in percentage as a function of the sample thickness for a frequency of 10GHz for titanium dioxide-wax and graphite-wax composites with different measurement set-up geometries.

Fig. 5 illustrates the dependence of sample thickness on permittivity. The results show that in general model 2 produces best results on both materials. Also it is shown that error in sample thickness produces higher error in imaginary component of permittivity of low loss material, this can be due to low absolute value of imaginary component of permittivity and therefore larger error. We obtained similar results for teflon which has low relative error for real component and highest relative error for imaginary component of permittivity, this can be also explained by low absolute value of imaginary component of permittivity and therefore larger error.

*Influence of probe size and frequency*

With simple test we also examined the effect of probe dimensions and operating frequency on the required the thickness of sample that can be used as semi-infinite sample. For reference we computed reflection coefficient for the geometry of semi-infinite sample. Then for finite thickness geometry we adapt the thickness of sample so that the reflection coefficient computed had the same value (on 6th decimal place) as reference reflection coefficient. Results for different probe dimensions can be seen in table 1.

**Table 1:** Influence of probe dimensions and frequency on electromagnetic field penetration for different materials

probe dimensions		frequency [GHz]	thickness [mm]	ε of material
a [mm]	b [mm]			
1,51	4,87	1	1120	10-0,01i
1,51	4,87	5	2070	10-0,01i
1,51	4,87	10	2080	10-0,01i
1,51	4,87	1	54	10-25i
1,51	4,87	5	18	10-25i
1,51	4,87	10	11	10-25i
0,225	0,84	1	20	10-25i
0,225	0,84	5	11	10-25i
0,225	0,84	10	7	10-25i
0,225	0,84	1	36	10-0,01i
0,225	0,84	5	182	10-0,01i
0,225	0,84	10	211	10-0,01i

As expected, for both probes material with higher dielectric losses needs lower thickness to be applicable as semi infinite sample. For the larger probe and relatively low-loss material the required thicknesses are substantial. The smaller probe shows the same dependence but required thicknesses are as expected much lower. But it is evident that operating frequency and probe dimensions are key factors when one wants to use semi-infinite geometry for measurement set-up.

Table 2 comprises probe dimensions, frequency of operation, true and obtained value of permittivity. The true value is permittivity used for calculation of reflection coefficient with parameters of  $d=50 \mu\text{m}$ ,  $L=500 \mu\text{m}$ , mode=12, model=2 and frequency=10 GHz, the obtained value is permittivity obtained by optimization algorithm from calculated reflection coefficient with parameters of  $d=0 \mu\text{m}$ ,  $L=500 \mu\text{m}$ , mode=12, model=2 and frequency at 10 GHz.

**Table 2:** Comparison of error in complex permittivity for used probes at 10 GHz

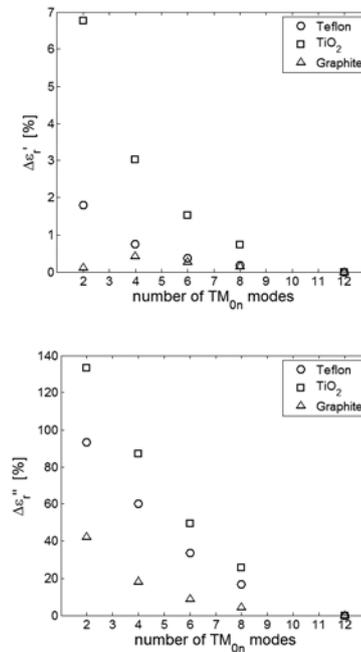
probe dimensions		frequency [GHz]	true value of $\epsilon$	obtained value of $\epsilon$
a [mm]	b [mm]			
1,51	4,87	10	10-0,09i	7,96-0,049i
0,225	0,84	10	10-0,09i	3,73-0,012i
1,51	4,87	10	2-0,003i	1,89-0,0028i
0,225	0,84	10	2-0,003i	1,60-0,0016i
1,51	4,87	10	40-25i	25,31-9,85i
0,225	0,84	10	40-25i	24,66-9,10i

Table 2 shows the difference between error in obtained permittivity with small and large probe. It is also seen that for low-loss materials small probe produces higher error in permittivity than large probe if the value of the air gap ( $d$ ) is not at the correct value (the difference is  $50 \mu\text{m}$ ). We obtain similar results for other values of air-gap uncertainty  $\Delta d$ . For high-loss materials both probes give similar error in obtained permittivity. The frequency has little effect on error of permittivity for both probes and the above conclusions are valid over the operating frequency range.

### Influence of $TM_{0n}$ modes

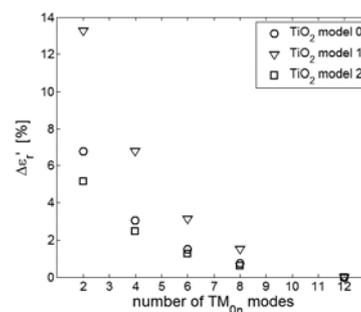
In our study we also analyzed the influence of number of used  $TM_{0n}$  modes with different geometry set-up. For reference value we used 12  $TM_{0n}$  modes. We observed different behavior on high-loss material as it has the smallest relative error in real part of permittivity with model 0 (Fig. 6) and highest relative error with model 1 and model 2. With model 1 and model 2 low-loss materials

has smallest relative error in real part of permittivity. One observes the opposite in relative error for imaginary part of permittivity for all three models (model 2 has the lowest relative error) where high-loss material has lowest error among all three materials. Again, higher relative error in imaginary component can be explained with low absolute value and therefore high relative error.



**Figure 6:** Error in real and imaginary component of permittivity in percentage as a function of a number of  $TM_{0n}$  modes for a frequency of 10GHz for different materials with measurement set-up geometry of model 0.

From data shown in figure 6 we can conclude that higher modes affect permittivity and should be used as many as possible. We used only 12 modes as reference and it is obvious that low number of used  $TM_{0n}$  modes contribute to measurement error.



**Figure 7:** Error in real component of permittivity in percentage as a function of a number of  $TM_{0n}$  modes for a frequency of 10GHz for titanium dioxide mixed with wax with different measurement set-up geometries.

Figure 7 shows that the optimum geometry set-up for titanium dioxide composite (relatively low-loss material) is dielectric terminated geometry (model 2). The same results came for imaginary component. Similar results were obtained for teflon and graphite composite. This is not surprising because in geometry of model 2 strong electric fields interact with sample material, thus giving good measurement results. And we also confirm that model 1 which has short-circuited termination has the worst results due to the boundary condition at the sample position. Electric fields in the sample are weak (magnetic fields are strong) and are approaching zero at the termination.

#### 4. Conclusion

In our study of open-ended coaxial probe system we analyzed effect of several key parameters of measurement set-up on measurement error of permittivity. The analysis showed that error in an air gap (error between actual and measured air gap) is clearly the single most important parameter as it produces highest error in permittivity among all studied parameters. Of the studied cases the least effect on error in permittivity was observable for low-loss materials with dielectric terminated measurement geometry.

Of other parameters size of the probe and operating frequency affect penetration of the field through the sample as large probe produces more field penetration in sample material than small probe. As expected, our calculations show that at high frequencies there is less penetration than at low frequencies for the case of high-loss materials. Just the opposite is seen for low-loss materials. Also uncertainty of the sample thickness does have some effect on the measured value of permittivity, but this parameter is much easier to control, especially for solid samples. Also, we analyzed the effect of the number of TM modes in calculations and clearly showed that with lower mode number the error can be significant. This is especially important since to obtain the permittivity one compares measured and calculated values of reflection coefficient. The accuracy of the obtained permittivity values is inherently limited by the accuracy of the calculation and this further strengthens the grounds for the use of the full-wave model with several TM modes over simpler models of open-ended coaxial probe.

When taken together our results show that the open-end coaxial probe system can be very problematic for solid samples and special effort must be applied to the air gap evaluation in order to get relevant values. Otherwise the method is limited to liquid or deformable materials where a gap can be eliminated. Further, it

can be concluded that dielectric terminated geometry (model 2) is best option among feasible measurement setup geometries for permittivity measurements and this is valid for both high- or low-loss materials.

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