

Kinetics of discharging arc formation

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Abstract: A scope of this paper is to present a mechanism of a discharge arc ignition in mechanically operated electric contacts in a gas mixture medium, such as the air. Introductory, the electric contacts are classified due to their mechanic and electric operation with given examples and corresponding most probable transient phenomena during their typical operation. In the first place, drawn arcs being metal vapour arcs of contact materials are most wearisome and destructive to the electric contacts, but mostly the discharging arcs are just preceding phenomenon to metal vapour arcs, and as such, they have indirectly the same effect on weariness of the electric contacts as the drawn arcs with intensity proportional to the arc current. A phenomenon of discharging arcs formation is discussed in this paper, which is an end result of throughout ionization and a throughout ionized path formation. The author's mathematical model calculates average kinetic energy of electrons in the non-homogeneous electric field due to an electron primary and secondary flow between two spherical electrodes. Exciting energy of gas molecules gained through electron impacts causes ionization of the molecules if the energy is high enough, but with the lower energy levels, dissociation of these molecules is carried out if they are at least two-atom molecules. Further on, the dissociated particles are associating in the other molecules, being also influenced by the electric field and so resulting in another processes of ionization and dissociation, and further on, recombination and association. There is continuous kinesis within the gas mixture, which gains a steady state mixture of the constituent gases, until the throughout ionization of one of the constituent gases at least is established by the increasing electric field throughout a space between the electrodes. So far, physics of this phenomenon deals with the electron kinetic energy and the energy of other energy carriers, such as photons and displacement current, in the electric field, and its transferring to molecules as the exciting energy causing their ionization and the ion recombination and the molecule dissociation, but the dissociated particles are part of chemical process, which is, together with their association in the newly produced compounds, dealt by chemism of this phenomenon. The new gaseous compounds have their own physics of their excitation in the electric field, and further on, the physics is followed by chemism of newly produced gases. Both of them, the physics and the chemism, results in kinetics of the throughout ionization formation and hence the discharging arc formation.

Keywords: discharging arc, gas throughout ionization, exciting energy, gas molecule dissociation, gas molecule ionization, gas chemism in electric field.

Kinetika nastanka razelektritvenih oblokov

Povzetek: Namen tega članka je predstavitev mehanizma vžiga razelektritvenega obloka v mehansko delujočih električnih kontaktih v zmesi plinov, kot je zrak. Uvodoma so električni kontakti razdeljeni glede na njihovo mehansko in električno delovanje s podanimi primeri in najbolj verjetnimi spremljajočimi tranzientnimi pojavi med njihovim tipičnim delovanjem. Na prvem mestu glede na obrabo in uničenjem električnih kontaktov so potegnjeni obloki, ki so obloki s kovinsko paro kontaktnih materialov. Toda večinoma so razelektritveni obloki predhodni pojav k oblokom s kovinsko paro in kot taki posredno enako učinkujejo na obrabo kontaktov kot potegnjeni obloki z jakostjo proporcionalno toku obloka. V tem članku je obravnavan pojav nastanka razelektritvenega obloka, ki je končni rezultat nastanka skozijske ionizacije in skozijske ionizacijske poti. S pomočjo avtorjevega matematičnega modela se izračunava povprečna kinetična energija elektronov v nehomogenem električnem polju zaradi primarnega in sekundarnega elektronskega toka med dvema kroglastima elektrodama. Vzbujevalna energija plinskih molekul, pridobljena s trki elektronov, povzroča ionizacijo molekul, če je energija dovolj visoka. Z nižjimi nivoji vzbujevalne energije se vrši disociacija – razdruževanje molekul, če se le-te vsaj dvoatomske. Nadalje se razdruženi delci združujejo v molekule drugih spojin in tako preidejo v druge procese ionizacije in disociacije ter nadalje rekombinacije in združevanja. V plinski mešanici, ki doseže stalno mešanico sestavnih plinov, obstaja nepretrgana kineza, dokler se pri večanju električnega polja ne vzpostavi skozijska ionizacija vsaj enega sestavnega plina plinske mešanice preko prostora med elektrodama. Fizika tega pojava obravnava kinetično energijo elektronov in energije drugih nosilcev, kot so fotoni in poljski tok, v električnem polju in njen prenos na molekule v obliki vzbujevalne energije, ki povzroča njihovo ionizacijo in rekombinacijo ionov ter razdruževanje molekul. Vendar razdruženi delci so del kemičnega procesa, ki je, skupaj z njihovim združevanjem v novo nastale spojine, obravnavan kot kemizem pojava. Nove plinaste spojine imajo svojo fiziko vzbujevanja v električnem polju, in nadalje, fiziki sledi kemizem novo nastalih plinov. Oboje, fizika in kemizem sestavljata kinetiko nastanka skozijske ionizacije in tako tudi nastanka razelektritvenega obloka.

Ključne besede: razelektritveni oblok, skozijska ionizacija plinov, vzbujevalna energija, disociacija plinskih molekul, ionizacija plinskih molekul, kemizem plinov v električnem polju.

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1. Introduction

Researching arcing between electric contacts, there are some differences between the transient phenomena due to contact mechanic and electric operation. When shifting contacts are making contact, their bouncing occurs, and hence drawn arcs, which are metal vapour arcs – vapour of contact materials, which usually are metals [1]. The drawn arcs occur also in holding mode with sliding contacts, since they slips in some kinds of their design (sliders, trolleys, slip rings) while they are holding electric contact. But, the drawn arcs are accompanying phenomenon at the operation of breaking electric current, especially in heavy duty operations, nevertheless, which kind of mechanic operation is used with the electric contact. So far, these differences between the electric contacts are over-viewed in Tab. 1.

Table 1: The clasification of electric contacts due to their machanic and electric operations in connection with the possible transient phenomena associated with their operations.

ELECTRIC CONTACTS						
TRANSIENT PHENOMENA	EXAMPLES	MOVEABLE				STACIONARY
		SHIFTING		SLIDING		
		perpendicular or angular direction to contact surface	parallel direction to contact surface	slipping	slipping & steady	
ELECTRIC OPERATIONS	MECHANIC OPERATIONS	holding	making	breaking	switching	
		break before make	make before break	break before make	make before break	
none	rivet on-off switch	holding	moving	moving & steady	switching	soldered welded wrapped faston screw bus
>discharging arcs at all operations >drawn arcs due to bouncing at making >drawn arcs at breaking	heavy duty relay telecommunication relay	break before make	make before break	break before make	make before break	
none	plug-socket connector	holding	slipping	slipping & steady	switching	
>discharging arcs at making and breaking >drawn arcs at holding >drawn arcs at breaking	slider trolley, slip ring	break before make	make before break	break before make	make before break	none
>discharging arcs at all operations >drawn arcs at making >drawn arcs at breaking	sampling switch commutator rotary switch with interruption rotary switch without interruption	holding	switching	switching	switching	

On the other hand, discharging arcs ignite by electric breakdown of throughout ionized surrounding gas medium between contact members [2], when they are in separated position, while they are closing at making operation, are opening at breaking operation or are opened still in switch-off position. The gaseous substance is surrounding gas medium, physically and chemically changed by electric field between the contact members.

The differences between the drawn arcs and the discharging arc are:

- in the time-depended electric current flow through gaseous substance between the contact members: in the drawn arcs, the current flow continues without interruption; but in the discharging arcs, it starts at zero and increases up to the arc ignition, or it is interrupted, reduced to zero and re-established through the arc ignition;
- in plasma particles, which depend on the gaseous substance between the contact members: in the drawn arcs, the ionized metal vapour of contact materials; but in the discharging arcs, the ionized gas of surrounding gas medium.

The discharging arcs would not be harmful by themselves to cause the contact wear by the involved electrons and the ions of the surrounding gas constituents, if the discharging arc did not invoke the metal vaporization and the ionization of the contact material followed by material disposition from one to another contact member. In this paper the kinetics, and thereby, the physics and the chemism of discharging arc are discussed.

2. The physics of the gas throughout ionization

The current between the separating contact members instantly falls towards zero value with the discharging arcs. A transient voltage appears due to time-derivative of the current, which extends to a breakdown voltage value of the medium – Fig. 1. The medium of the discharging arc is the existing ionized gas from the surrounding space.

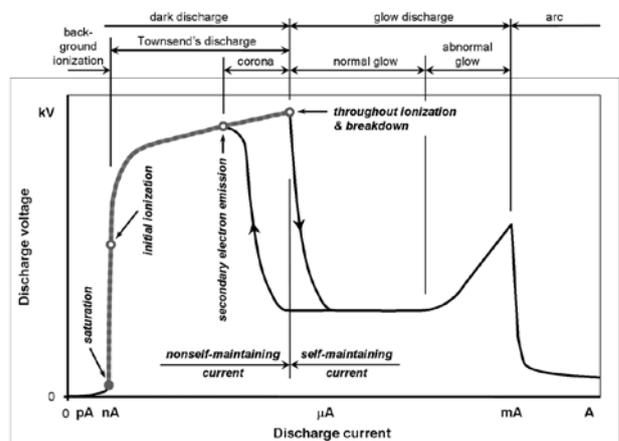


Figure 1: The principle electric discharge UI characteristic [3] with the range hereafter dealt in this discourse.

With the increasing transient voltage, the dielectric breakdown of the insulating gas occurs, and due to it, the electric current increases. The kind of discharge, which follows, depends on the current through the gas: the dark discharge, the glow discharge or the discharging arc, either stable or unstable, the latter one resulting in sparking. The separating contact rivets in some opening distance are substituted by a spark gap of two spherical electrodes to research the discharging arc formation. Therefore the mathematical model of the spark gap was developed to study the electric field and the ionizing process in the gap. The electric load is an air coil in this mathematical model, replaced by a conceptual circuit equivalent in very simplified way.

The spark gap consists of two spherical electrodes with the same radius r_0 , separated by the distance d_{sur} between their surfaces and the gas medium around them. An anode is positively charged and a cathode has a negative charge of the same absolute amount of charge in the first stage of the arc development, when there is no charge in the space nearby the cathode, so far without a cathode layer. The cathode is earthed so that a positive charge flows from the cathode to the earth. Due to the mutual influence of the anode and the cathode charges, the equivalent point charges of the anode and the cathode, $+Q_1$ and $-Q_1$, lie in the eccentric positions in the relevant spheres – Fig. 2. In the cathode layer shown in Figs 2 and 3, when it comes to existence, there are a positive ions layer and an electrons layer. Since the positive ions layer is closer to the cathode than the electrons layer, a distributed additional negative charge is induced just beneath the cathode surface. The integral of the distributed positive charge of the ions layer over volume, the integral of the distributed charge of the electrons layer over volume, and the integral of the distributed additional negative charge of the cathode are equal by their absolute values. Due to the signs assigned to these charges, the sum of these integrals equals to the integral of the distributed additional negative charge of the cathode when radii of the ions layer and the electrons layer are limiting to the cathode radius, and so far the integrated additional negative charge of the cathode is substituted by the equivalent point charge $-Q_2$ in the cathode centre.

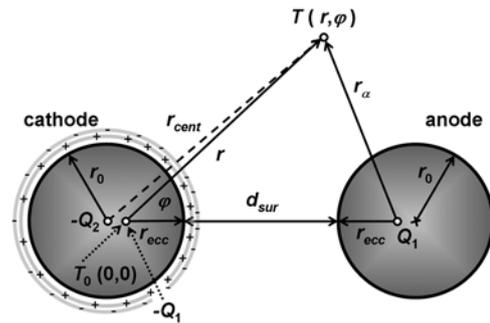


Figure 2: The geometrical drawing of the cathode with the layer of spatial distributed electric charge of integrated values of $+Q_2$ and $-Q_2$ around the cathode, the anode and the distances between them in $r-\varphi$ coordinate system.

The mathematical model in spherical coordinates (r, φ, Φ) is simplified to two dimensional space $(r, \varphi, \Phi = \pi/2) \rightarrow (r, \varphi)$ because the electric field is rotary symmetrical. Hereafter, to avoid misunderstanding, the usage of Cartesian coordinates refers strictly to the plane (r, φ) , and not to three dimensional space. The eccentric positions of the equivalent point charges are defined by eccentric radius r_{ecc} as it is shown in Fig. 2, and further on, the eccentric position of the cathode equivalent point charge $-Q_1$ is the zero point of the coordinate system. A potential U of any point $T(r, \varphi) = T(r, r_\alpha, r_{cent})$ is an algebraic sum of the partial potentials caused by the anode and the cathode charges since the potential is scalar value. It is defined by the following equation in the bi-radial coordinates r, r_α plus dependent coordinate r_{cent} as follows [4]:

$$U(r, r_\alpha, r_{cent}) = \frac{Q_1}{4 \cdot \pi \cdot \epsilon} \left(-\frac{1}{r} + \frac{1}{r_\alpha} + \frac{1}{r_{ecc}} - \frac{1}{r_{ecc} + d_{sur}} \right) + \frac{Q_2}{4 \cdot \pi \cdot \epsilon} \left(-\frac{1}{r_{cent}} + \frac{1}{r_0} \right) \quad (1)$$

where Q_1 and Q_2 are absolute values of point charges. Variables of this equation are coordinates, as already mentioned, r, r_α, r_{cent} as shown in Fig. 2.

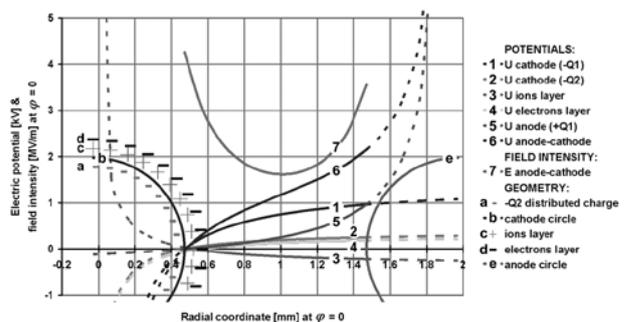


Figure 3: The analysis of the electric potentials and the electric field intensity due to the anode, the cathode and the spark gap geometry.

Since the cathode is earthed, its surface potential is defined as zero, which is also zero at the following point on its surface:

$$r = r_{ecc} \wedge r_a = r_{ecc} + d_{sur} \wedge r_{cent} = r_0 \Leftrightarrow U = 0 \quad (2),$$

hence the geometric constants in Eq. (1). The electric charge Q_1 is modelled due to the primary electric field and the primary electron emission, but Q_2 due to the spatial distributed electric charges in the cathode layer as in Fig. 2 maintaining both the primary and the secondary electron emission.

Solving the geometry of the system in Fig. 2, following geometric quantities are obtained: firstly, the surface radius of the cathode r_{surk} depending on the angle coordinate φ and used later in this discussion, and secondly, the radial coordinates r_a and r_{cent} in terms of the polar coordinates r and φ , which transform Eq. (1) in the function $U(r, \varphi)$ – the electric potential depending on the polar coordinates.

Between the anode and the cathode a certain voltage is applied, as follows:

$$r = r_{ecc} + d_{sur} \wedge \varphi = 0 \Rightarrow \quad (3)$$

$$\Rightarrow U_{ak} = \frac{2 \cdot Q_1}{4 \cdot \pi \cdot \epsilon \cdot r_{ecc} \cdot \left(1 + \frac{r_{ecc}}{d_{sur}}\right)} + \frac{Q_2}{4 \cdot \pi \cdot \epsilon \cdot r_0 \cdot \left(1 + \frac{r_0}{d_{sur}}\right)}$$

if there are both the primary and the secondary electron emissions, else Q_2 is zero.

If the electric field intensity, causing only the primary electron emission when weak, is strong enough to accelerate the electrons to ionize the gas molecules near the cathode, the secondary electron emission from the cathode arises by the gas ions hitting it. So far, a voltage drop over the narrow gap of $\lambda \cdot N_\lambda$ (an average electron free path multiplied by a natural number, explained later on) around the cathode, but not necessarily the whole around, with no secondary electron emission present is equal to the must-be value needed for gas molecule ionization. By meeting this condition, the secondary electron emission begins and the electric field intensity is being increased by the electric charge Q_2 , which causes both primary and secondary emissions increasing. With the phenomenon of the secondary electron emission, corona is associated burning near the cathode. The electric charge Q_2 increases until the electric breakdown takes place. Beyond the point of the electric breakdown the presented mathematical model is not valid any more.

The electric charges Q_1 and Q_2 depend on the increasing anode-cathode voltage U_{ak} and infinite set of kinet-

ic energy values around the cathode sphere obtained by electron while moving from the cathode surface, determined by the radii values r_{surk} up to the distance of

$$\left(r_{surk} + \frac{dr_{surk}}{dr_0} \cdot \lambda \cdot N_\lambda \right),$$

in the electric field of $\vec{E}(r, \varphi, Q_1, Q_2 = 0)$ as there is only the primary electron emission. The parameter N_λ is natural number telling at which distance from the cathode the collision phenomenon begins: up to $1 \cdot \lambda$ there is no collisions, in the interval from $1 \cdot \lambda$ inclusive and up to $2 \cdot \lambda$ there are the first collisions. So far, this parameter is taken as $N_\lambda = 2$ in this discussion. On the other hand this number also defines an interruption in the ionization process carried out by consecutive collisions process: the continuous ionization process is interrupted if only every N_λ -th collision causes the molecule (atom) ionization – every second collision in particular case of choice in presented model. On the basis of these preassumptions used in the model, the charges Q_1 and Q_2 are determined.

The electric field intensity is derived from the electric potential function $U(r, \varphi)$ by using the Hamilton operator:

$$\vec{E} = -\nabla \cdot U(r, \varphi) = -\text{grad}(U(r, \varphi)) = -\left(\hat{1}_r \cdot \frac{dU(r, \varphi)}{dr} + \hat{1}_\varphi \cdot \frac{dU(r, \varphi)}{r \cdot d\varphi} \right) \quad (4).$$

The eccentric radius r_{ecc} as it is defined in Fig. 2, is established by finding the closest equipotential surface to the sphere with the radius r_0 , which is its geometric radius. The equipotential surface is not exactly the sphere but is near enough not to cause any significant error. Establishing the eccentric radius r_{ecc} as it is defined in Fig. 2, the iterative procedure is used, based on similarity to the mathematic inversion of one point charge on the sphere with the radius of r_0 , known as Kelvin's transformation also, while a coefficient K_{eq} is determined to find the best approximated equivalent substituting two equal metallized and charged spheres (the anode and the cathode) with two eccentric points of opposite electric charge Q_1 inside these spheres:

$$r_{ecc} = \frac{d_{sur}}{2} \cdot \left(\sqrt{1 + 4 \cdot \frac{r_0}{d_{sur}}} - 1 \right) \cdot K_{eq} \quad (5)$$

at the charge Q_2 being zero [2]. So far, the equipotential surface of zero potential fits the cathode sphere better than 2.06 % with 95 % statistical confidence when the input data are $r_0 = 0.5$ mm and $d_{sur} = 1$ mm, but the zero cathode potential uncertainty is 1.16 % of the anode voltage by three iterations and with 95% statistical confidence.

When calculating the mean free path of electrons, quite large uncertainty is associated to this calculation. To demonstrate quite large deviations from the mean value some probability distributions are considered, and further on molecule radii discussed. The average velocity of an accelerated electron is calculated due to its exponential probability distribution and due to the final velocity value of the electron accelerated movement. The average velocity of an air gas molecule (O_2 or N_2) is established due to the Maxwell – Boltzman probability distribution. Further on, the average free path of the electron up to collision with the gas molecule due to its exponential probability distribution is derived as its expectation value and is [5]:

$$\lambda = \frac{R \cdot T}{\sqrt{1 + \frac{v_{m_avg}^2}{v_{e_avg}^2}} \cdot \pi \cdot d^2 \cdot N_A \cdot p} \quad (6)$$

where the quantity d is the sum of molecule and electron radii, defined as a collision diameter:

$$d = r_m + r_e \quad (7)$$

R is the gas constant, p and T are gas pressure and temperature, and v_{m_avg} and v_{e_avg} are average velocities of molecules and electrons respectively.

A covalent radius is the nominal radius of the atoms of an element when covalently bound to other atoms, as deduced through the separation between the atomic nuclei in molecules. In principle, the distance between two atoms that are bound to each other in a molecule (the length of that covalent bond) should equal the sum of their covalent radii. In the previous papers [2] the oxygen atom covalent radius of 68 pm was taken into account [6] and hence the molecule radius of 133.4 pm, and so far, the relevant value of the electron kinetic energy was estimated rather too low, which caused breakdown voltage values being estimated very high. Using Van der Waals radii would result in still lower values of the electron energy and higher breakdown voltage estimates. Therefore the empirical covalent radii (oxygen: 60.4 pm, nitrogen: 54.9 pm), the relevant covalent bond lengths of diatomic molecules (oxygen: 121.0 pm, nitrogen: 110.0 pm) are used afterwards [7, 8]. The molecule of diatomic gases is approximated as a rod with rounded ends and the collision area is obtained by this rod projection in the direction of the electron movement using the mean value of the collision area due to the molecule rotation. The molecule collision radii (oxygen: 81.3 pm, nitrogen: 73.9 pm) are calculated due to the mean value of the collision area. Since in the vicinity of less than $\lambda \cdot N_A$ to the cathode there is no impacts of electrons, the average kinetic en-

ergy of the electron travelling along the average free path is taken into account at the radii of

$$r > r_{surk} + \frac{dr_{surk}}{dr_0} \cdot \lambda \cdot N_A \text{ with } N_A = 2 \text{ in the presented model:}$$

$$W_{ek} = e \cdot E(r, \varphi) \cdot \lambda \quad (8)$$

whilst the electric field intensity and the average free path are collinear vectors.

When the electron collided with the gas molecule, its kinetic energy from Eq. (8) and the initial electron kinetic energy carried on from the previous collision W_{car_on} are together transferred to the molecule by impact as exciting energy of the molecule:

$$W_{exm} = W_{ek} + W_{car_on} \quad (9)$$

In this discourse the term of excitation, and hence the exciting energy, is used, according to some scientific terminology, in the unconventional way. The excitation as a general term is an elevation in energy level above an arbitrary baseline energy state of an atom or a molecule without causing any changes in its charge on the whole (ionization, electron attachment) or any chemical changes (molecule dissociation). But hereafter, the excitation, and hence exciting energy, means firstly an increase in energy level up to the levels of ionization and dissociation, if applicable, secondly their accomplishment, and thirdly causing the changes in energy of newly begotten particles – the elevation in energy level with ions or dissociated atoms, or changes in kinetic energy of free electrons, if there are any involved in the excitation process. Namely, the excitation is not obtained by the electron collisions only, but also by photon impacts and through a displacement current effect. But, the outcoming excited particles are considered to have a short lifetime, and afterwards produce the photon, which further on cause the excitation of another atom or molecule. So far, we will not deal with the changes in energy level of outcoming particles (ions, dissociated atoms), but the part of the exciting energy beyond the ionization or the dissociation energy is attributed to the involved electrons, if it is applicable, but otherwise to the photons with the same end effect, as in the case of the involved electrons, but with the mass equals to zero. In both cases this energy is carry-on energy.

Due to the function of the electric field intensity depending on the radius there are three ranges between the electrodes: a highly ionized range, a partly ionized range and a non-ionized range – Fig. 4.

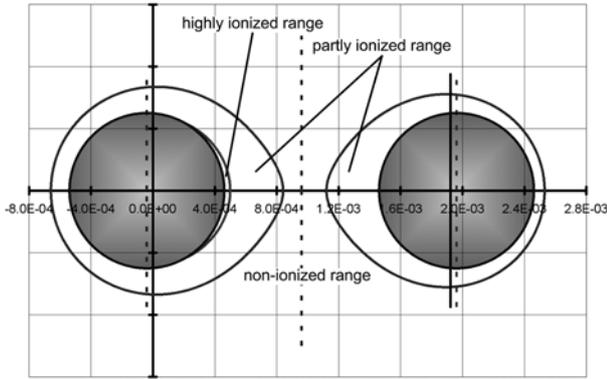


Figure 4: The ranges due to ionization degrees of nitrogen (N) in the midst of the air medium in the electric field – the ionization degrees refer to nitrogen atoms only.

The **highly ionized range** is in the vicinity of the each electrode because nearly every collision causes such exciting energy in the molecule that its ionization occurs due to very high kinetic energy of the electrons, and in the one of the next moments the ionization is followed by the recombination. From the viewpoint of kinetic energy of colliding particles before and after an inelastic collision, the kinetic energy lost by the collision is mainly consumed by some other process – in discussed case as the ionization energy, and further on as the dissociation energy, whereas the kinetic energy before the collision is the exciting energy being stored as the kinetic energy of an electron. Taking into account this loss defined by the difference ($W_{before} - W_{after}$), a fraction of the kinetic energy loss is introduced as

$$\frac{W_{before} - W_{after}}{W_{before}} = \frac{W_{icon}}{W_{exm}} = \left(\frac{m_m}{m_e + m_m} \right)$$

at a total inelastic collision, and so the exciting energy of the molecule must be at least [9]:

$$W_{exm} \geq W_{ion} \cdot \left(1 + \frac{m_e}{m_m} \right) \quad (10)$$

to cause ionization. The exciting energy of the whole amount of moving electrons with their kinetic energy, is divided between the ionization as the P_i part and the recombination as the $(1-P_i)$ part.

After the **ionization collision**, the average carry-on kinetic energy per electron, carried on by the one colliding and by the one emitted is:

$$W_{car_on} = \frac{W_{exm} - W_{icon} \cdot \left(1 + \frac{m_e}{m_m} \right)}{2} \cdot P_i \quad (11).$$

After the **recombination collision**, since the collision is totally inelastic, and it consumes the whole exciting energy received by the electron, and further on, one part of recombined molecule energy ΔW_{rec} is emitted (for instance as emitted radiation energy, where the photon is energy carrier) and further consumed by ionizing molecules in continuous process:

$$\Delta W_{rec} = W_{icon} \cdot (1 - P_i) \quad (12),$$

but the other part is transformed to thermal energy, which K -part is conveyed to surroundings by thermal conduction, convection and/or radiation, but $(1-K)$ -part of it causes the molecule temperature rise above the ambient temperature for the increment:

$$\Delta T = \frac{2 \cdot (W_{exm} - W_{icon})}{3 \cdot k} \cdot (1 - P_i) \cdot (1 - K) \quad (13).$$

So far the average temperature of the gas in the neighbourhood of the cathode and the anode rises and the temperature of the each electrode increases too when the molecules bump at it. The parameter $K=99.8\%$ in Eq. (13) defines the percentage of the energy in this equation, conveyed and conducted to the cathode, the anode, and further on, to the ambient by a natural or a forced cooling. Due to the temperature increment of Eq. (13), the average kinetic energy of the gas molecules increases, and hence the average molecule velocity.

Next to this range, the **partly ionized range** exists up to the point where no ionization occurs. In this range, besides the ionization collisions, the **dissociation collisions** are happening with the exciting energy of the molecule equal to:

$$W_{icon} \cdot \left(1 + \frac{m_e}{m_m} \right) > W_{exm} \geq W_{diss} \cdot \left(1 + \frac{m_e}{m_m} \right) \quad (14).$$

In this case, the exciting energy of the gas molecule causes the **dissociation** of the two-atom molecule into two gas atoms. This collision is partly inelastic and it consumes the dissociation energy. The remaining kinetic energy of the colliding electron is carried on by the same electron:

$$W_{car_on} = W_{exm} - W_{diss} \cdot \left(1 + \frac{m_e}{m_m} \right) \quad (15),$$

and further on, it increases because the electron passes the next average free path before the next collision – Eq. (8).

Towards the gap centre, the **non-ionized range** begins, where the dissociation collisions and also the **unaffected collisions** are present. If the exciting energy of the

molecule is lower than the dissociation energy of the gas molecule:

$$W_{diss} \cdot \left(1 + \frac{m_e}{m_m} \right) > W_{exm} \quad (16),$$

the colliding electron has no effect on the gas molecule. The kinetic energy of the electron after the collision is the same as before, which is:

$$W_{car_on} = W_{exm} \quad (17).$$

Although there is a sequence of all mentioned phenomena, we could not consider there are any pure ranges such as an ionization-recombination, a dissociation or an unaffected range.

Knowing the electric field intensity at the cathode surface at $\varphi = 0$ and its temperature, the following conductive current densities are calculated: the current density of the field emission and the current density of the thermionic emission. Both of them are the active current densities.

If an alternating electric field is applied, as in this particular case caused by a sinusoid of the anodecathode voltage with the amplitude of 20 kV and the frequency 5 kHz being interrupted by the breakdown after achieving the throughout ionization voltage, the displacement current occurs. The displacement current is defined by its density [4], and the latter is, likewise the electric field intensity, depending on time, therefore both of them can be represented by phasors in complex plane. An angle δ between the displacement current density phasor and the phasor of time-derivative of the electric field intensity is defined by the complex value of the relative permittivity. The displacement current density is vector, collinear to the time-derivative of the electric field intensity vector multiplied by absolute value of relative permittivity ϵ_r and the permittivity ϵ_0 of vacuum, and hence collinear to the electric field intensity vector. The relative permittivity is complex scalar constant defined by its absolute value and by losses angle δ , where the dielectric losses of the gas are defined as the imaginary part of the complex value of the relative permittivity, so they are associated with sinus function of the losses angle. The active displacement current causes the excitation of the molecules and the phasor of this current density is in phase with the electric field intensity phasor in the complex plane. The reactive displacement current vector is a capacitive current, so far, its phasor is perpendicular to the electric field intensity phasor, but the relevant vectors are collinear. Because the active displacement current causes the excitation of the molecule in the volume between the cathode and the anode, the ac-

tive displacement power is the integral of the product of the phasors of the active displacement current and of the electric field intensity throughout the volume relevant to one molecule that is the gas molecule itself and in the hollow volume around it, which is

$\frac{V_{mol}(p, T)}{N_A}$ (a volume of one mol divided by the Avogadro number), under the relevant thermodynamic conditions of the gas. Further on, the active displacement energy of one molecule is defined by:

$$W_{Dact} \approx \epsilon_r \cdot \epsilon_0 \cdot \frac{dE(r, \varphi, t)}{dt} \cdot E(r, \varphi, t) \cdot \frac{V_{mol}}{N_A} \cdot \frac{\lambda}{v_{e_avg}} \cdot \sin \delta \quad (18).$$

The losses angle δ is defined by the ratio of the volume of one molecule V_m and its relevant part of empty space belonging to it:

$$\tan \delta = \frac{1}{\frac{V_{mol}(p, T)}{V_m \cdot N_A} - 1} \quad (19).$$

The mathematical model of the electric discharge in gases has to take into account both, the kinetic energy of the electrons, the energy of photons and the energy of the displacement current. The electron kinetic energy is partly transferred to the gas molecule by the electron impact, and causes the ionization or the dissociation, discussed heretofore. In this case, the ionization is considered as the impact ionization although it is more probable that the ionization is done through the excitation of the gas molecule on its higher energy level [5]. The dissociation of the two-atom molecule just cannot be carried out directly by the electron impact due to the large difference of the electron mass and the dissociated atom mass. The dissociation is completed by the exciting energy of the two-atom molecule due to the impact energy when raised in such extend that the dissociation energy level is achieved. This is the dissociation due to the conductive and the convective current. The displacement current energy also affects the gas molecules, and also causes their ionization and their dissociation. Because it has no carriers, the ionization and the dissociation are caused by the excitation of the gas molecule with no impact, but only due to the displacement current. All these processes: the impact ionization, the dissociation – both due to the conductive and the convective current, the ionization and the dissociation – due to the displacement current, have the same mechanism of being completed – raising the molecule energy on its higher energy level, and afterwards the accomplishment of the process. Therefore the kinetic energy and the displacement energy

Eq. (18) are summarized in the exciting energy of the gas molecule, which is the active energy:

$$W_{exm}(r, \varphi, t, p, T, W_{ion}, W_{diss}, m_m) = e \cdot E(r, \varphi, t) \cdot \lambda(p, T) + \quad (20),$$

$$+ W_{car_on} \left(W_{ion} \cdot \left(1 + \frac{m_e \mathbf{v} \cdot \mathbf{0}}{m_m} \right), W_{diss} \cdot \left(1 + \frac{m_e \mathbf{v} \cdot \mathbf{0}}{m_m} \right) \right) +$$

$$+ \varepsilon_r \cdot \varepsilon_0 \cdot \frac{dE(r, \varphi, t)}{dt} \cdot E(r, \varphi, t) \cdot \frac{V_{mol}(p, T)}{N_A} \cdot \frac{\lambda(p, T)}{v_{e_avg}} \cdot \sin \delta$$

where the mass of the impacting particle has the value of m_e if it is an electron, or has the value of zero if it is a photon.

Due to carry-on energy the exciting energy of the gas molecule has its minimal value and its maximal value, as follows:

$$W_{exm_mn}(r, \varphi, t, p, T, m_m) =$$

$$= e \cdot E(r, \varphi, t) \cdot \lambda(p, T) + \quad (21),$$

$$+ \varepsilon_r \cdot \varepsilon_0 \cdot \frac{dE(r, \varphi, t)}{dt} \cdot E(r, \varphi, t) \cdot \frac{V_{mol}(p, T)}{N_A} \cdot \frac{\lambda(p, T)}{v_{e_avg}} \cdot \sin \delta$$

$$W_{exm_mx}(r, \varphi, t, p, T, W_{ion}, W_{diss}, m_m) = \quad (22).$$

$$= e \cdot E(r, \varphi, t) \cdot \lambda(p, T) +$$

$$+ (W_{ion} - W_{diss}) \cdot \left(1 + \frac{m_e}{m_m} \right) +$$

$$+ \varepsilon_r \cdot \varepsilon_0 \cdot \frac{dE(r, \varphi, t)}{dt} \cdot E(r, \varphi, t) \cdot \frac{V_{mol}(p, T)}{N_A} \cdot \frac{\lambda(p, T)}{v_{e_avg}} \cdot \sin \delta$$

The minimal value is very obvious, but the maximal value is numerically determined by the mathematical model. The exciting energy values, at the one and the same point (r, φ) , are lying between these two values concerning the carry-on energy probability distribution, which is the square probability distribution. The probability distributions of the other quantities in Eq. (20) would contribute their shares to the exciting energy probability on the whole, but it is reasonable that these other contributions are considered as a part of uncertainty calculation.

With this phenomenon especial curves and surfaces are defined, which are equi-exciting energy curves and surfaces. The **equi-exciting energy line (or surface)** in the electric field within the medium of the particular gas is the set of points at which the gas molecules are exposed to the same exciting energy at the same exciting energy probability:

$$C_{eqex} = \{ T_i(r, \varphi) \Leftarrow W_{exm}(r, \varphi, t, p, T, W_{ion}, W_{diss}, m_m) = \quad (23).$$

$$= constant \wedge P_{Wexm} = constant \}$$

For various constants the family of the curves (surfaces) is achieved. Among these curves (surfaces), corresponding to Eq. (23) there is one most important, and it is a border curve between the ionized range and the non-ionized range in Fig. 4, and further on also in Fig. 10, considering at least some ionization is carried out by photons:

$$C_{eqex_ion} = \{ T_i(r, \varphi) \Leftarrow \quad (24).$$

$$\Leftarrow W_{exm_mx}(r, \varphi, t, p, T, W_{ion}, W_{diss}, m_m) = W_{ion} \}$$

So far, the **ionization border line (or surface)** is a particular equi-exciting energy line (surface) within the particular gas distinguishing between the area (space) of the ionized gas on one side of this curve and the area (space) on the other side of the curve predominately occupied by its neutral molecules and atoms (also dissociated if relevant) with no ionized particles. The whole exciting energy probability distribution domain is below the value of the ionization energy of the gas molecule, and the ionization occurs just at the upper domain border, hence the probability that the exciting energy is lower than the ionization energy of the particular gas is one ($P_{Wexm} = 1$) in the non-ionized range in Fig. 4:

$$R_{non_ion} = \{ T_i(r, \varphi) \Leftarrow \quad (25).$$

$$\Leftarrow W_{exm_mx}(r, \varphi, t, p, T, W_{ion}, W_{diss}, m_m) < W_{ion} \}$$

The ionization border line is an outer contour of both the highly and the partly ionized ranges in Fig. 4.

If the anode-cathode voltage increases, the non-ionized range in Fig. 4 narrows, and the partly ionized range from the cathode side touches the partly ionized range from the anode side, and the ionized path between the electrodes arises throughout the gas gap, the dielectric breakdown occurs and the electric discharge arc takes place.

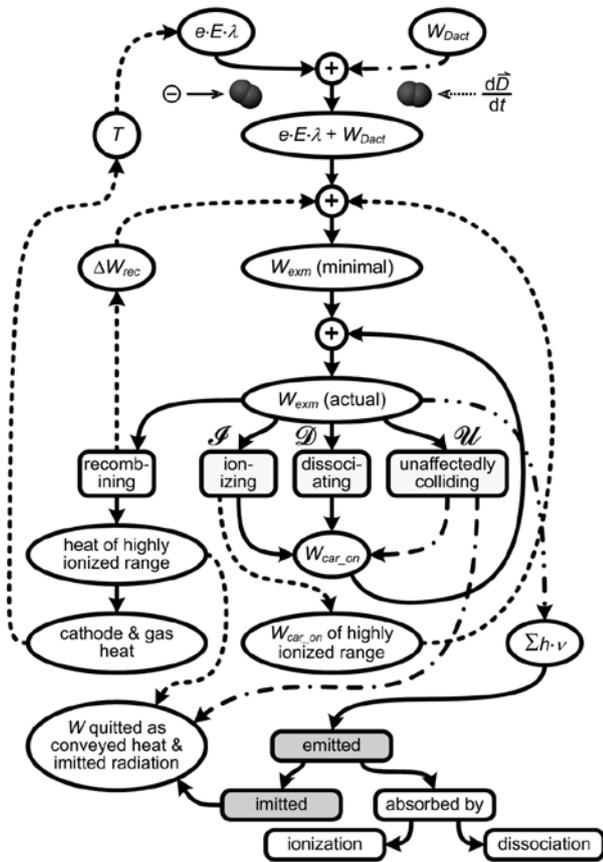


Figure 5: The physics of arising the discharging arc in the two-atom molecule gas.

Hereafter, the mathematical algorithm, describing the phenomenon of arising the discharging arc in the stage of building the ionized path through gas gap up to a throughout ionization, was computerised to get a dynamic model of this phenomenon as shown in Fig. 5.

There are three kinds of electric current density, beginning with the one already discussed:

- the displacement current density, as discussed;
- the conductive current density, which is in phase with electric field intensity, appears in consequence of the cold electron emission, due to the electric field effect, and due to the thermionic electron emission, whose relevant current density is, to the large degree, independent on time, but time-dependent part is in phase with the electric field intensity;
- the convective current is Laplacian of the electric potential $U(r, \varphi, t)$, which is direct or alternating quantity; since the Laplacian is zero in the state before the electric breakdown, the convective current is zero too.

The total cathode current is achieved by integration of the vector sum of all contributing current densities over

the cathode surface in spherical coordinates (r, φ, ϕ) taking into account the rotary symmetry of the electric field and system geometry:

$$I_k = 2 \cdot \int_{\phi=0}^{\phi=\pi} \int_{\varphi=0}^{\varphi=\pi} \left[\vec{j}_D \left(\frac{d\vec{E}}{dt} \right) + (j_E(|\vec{E}|) + j_T(T_k, |\vec{E}|)) \right] \cdot \frac{\vec{E}(r, \varphi, t)}{|\vec{E}(r, \varphi, t)|} + \vec{j}_{conv}(\vec{E}) \cdot r^2_{surk}(\varphi) \cdot d\varphi \cdot \sin \phi \cdot d\phi \quad (26)$$

having in mind the quantities depended on the electric field intensity depends indirectly on (r, φ) and time, and moreover each of them depends on the cathode temperature.

The shortest field line between the spherical electrodes of the opposite charges is the shortest surface-to-surface distance between the spheres. In this case the angle φ is zero. The vector of electric field intensity has only radial component since the angular component is zero. Hereafter the mathematical model deals with the electric field and the phenomena associated with it in this particular direction. Therefore, the radial coordinate in this direction $\varphi = 0$ is named as r_x – radius in the x -direction of the (r, φ) plane.

When discussing the electrical breaking contact, its contact members are the electrodes. The distance between them increases from zero, when the breaking contact still holds the closed position, up to the maximum value. In the model, the distance of 1 mm is used because the Paschen law minimum is being avoided.

3. The chemism the gas throughout ionization

The whole kinetics of the discharging arc formation, and hence of the throughout ionization of a gas mixture medium, continuing to the electric breakdown, and further on, followed by the discharging arc ignition in that medium, consists of its physics, discussed heretofore, and its chemism – the chemism of the discharging arc formation in any gas mixture as well as in any two-atom molecule gas even of chemical single-element. In the process of the gas medium ionization, each of the gases is ionized and afterwards ions are recombined, and multi-atoms molecules are dissociated and later their atoms associate in another molecules, all happening fluently and continuously forming the ionized ranges ending finally in the throughout ionization of the gas in the gap between electrodes if only the electric field in the gap is sufficient for it. And this part of the kinetics of the throughout ionization formation, which consequently leads to the electric breakdown and to the discharging arcs ignition, is the chemism of this phenomenon. When dealing with air as a gas me-

dium, all constituents must be taken into account: oxygen, nitrogen, argon, carbon dioxide etc. In this paper, only oxygen and nitrogen, as major constituents of air, are considered when modelling the throughout ionization of the air gap.

In the case of the O_2 - O - O_3 gas mixture, as well as in the case of the N_2 - N - NO_2 gas mixture, there are three kinds of gas and its ions in each mixture, and hence the physics of the processes are defined in the following way:

- the ionization by the set of the transfer functions $I = \{f_{ion}, g_{ion}, h_{ion}\}$ for each gas in the mixture respectively;
- the dissociation by the set of the transfer functions $D = \{f_{dis}, g_{dis}, h_{dis}\}$ for each gas in the mixture respectively;
- the unaffected collisions by the set of the transfer functions $U = \{f_{unaff}, g_{unaff}, h_{unaff}\}$ for each gas in the mixture respectively.

The sum of the f -functions is one whenever and wherever in the space, and likewise the sum of the g -functions and the sum of the h -functions for each kind of compounds: the oxygen and the nitrogen compounds separately, and these sets are also different for each of them. The chemical process of the oxygen compounds mixture is shown in Fig. 6, but the chemical process of the nitrogen compounds mixture in Fig. 8, both followed by the relevant results and discussion.

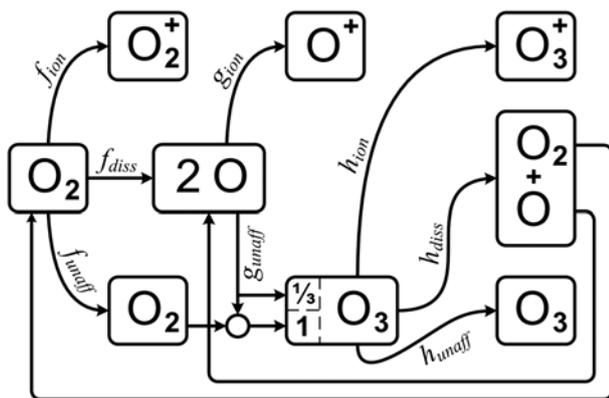


Figure 6: The chemical process of oxygen compounds in the electric field.

Yields of the outcoming gases along the shortest path between the cathode and the anode are pre-sented graphically in Fig. 7 in the moment of the throughout ionization of O_3 molecules, which is the end of the discharging arc formation in the air followed by the electric breakdown and by the discharging arc itself.

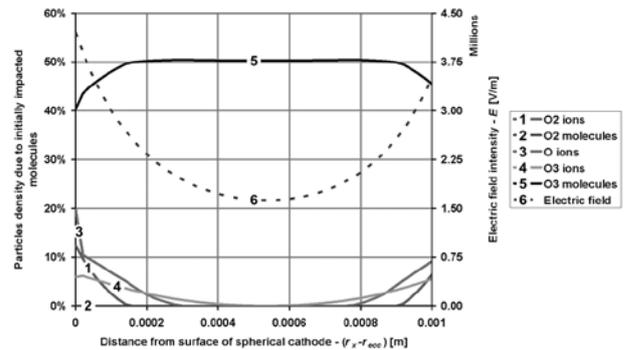


Figure 7: The yields of the chemical process of oxygen compounds of the air in the electric field along the shortest path between the electrodes.

The ratios between the oxygen compound particles densities, which are the output of the chemical process, and the initially impacted oxygen molecules density, which is the input of the chemical process, are the yields of the chemical process output, indexed as in Fig. 7, are as follows: $y_1(O_2 \rightarrow O_2^+)$, $y_2(O_2 \rightarrow O_2)$, $y_3(O_2 \rightarrow O^+)$, $y_4(O_2 \rightarrow O_3^+)$ and $y_5(O_2 \rightarrow O_3)$. The chemical process produces the mixture of the oxygen compound particles, which maintains its structure ratios despite its continuous kinesis: ionizing, recombining, dissociating and associating the molecules, the atoms and the ions respectively, whatever is appropriate for each kind of particle, within its steady state structure.

The model of the chemism of the discharging arc formation in oxygen presumes that the whole amount of the O atoms, after the O_2 molecules dissociation, is combined to ozone in one or another way. These atoms are primarily associated to the unaffected O_2 molecules, but there can be some leftover amounts either of the O atoms or the O_2 molecules. If there are the O atoms leftover, they are combined with themselves to form ozone. On the other hand, if there are the O_2 molecules leftover, they are outcome of the process by the yield of y_2 as unaffected and not combined to ozone, which is shown in Fig. 7 as the result of this (and only this) particular case, which is zero throughout the gap.

The chemical process with the O_2 - O - O_3 gas mixture is the same from the standpoint of oxygen compounds structure whether there is pure oxygen involved in it or there is some other gas in the mixture as the nitrogen in the air. But, it is not the same with nitrogen since the chemical process of the pure nitrogen in the electric field gives the N_2 -N mixture including their ions, but the chemical process of nitrogen in the air produces the N_2 -N- NO_2 mixture including their ions and dissociated particles. Hereafter the N_2 -N- NO_2 mixture is dealt with. It is presumed that there are no volatile organic compounds in the standardized (unpolluted) atmosphere, and hence

the dissociation of NO_2 is defined as $\text{NO}_2 \rightarrow \text{NO} + \text{O}$, and further oxygen atom reacts with O_2 molecule from the air $\text{O}_2 + \text{O} \rightarrow \text{O}_3$ and produced ozone, and once again oxidizes nitrogen monoxide $\text{NO} + \text{O}_3 \rightarrow \text{NO}_2 + \text{O}_2$ returning the O_2 molecule into the air, so effectually the O_2 molecules do not need to be exchanged from and into the air, but is produced and consumed inside the chemical sub-process following the dissociation of the NO_2 molecules – Fig. 8. The NO_2 molecules, which resulted from this sub-process, undergo the electric field and are ionized, dissociated or unaffected.

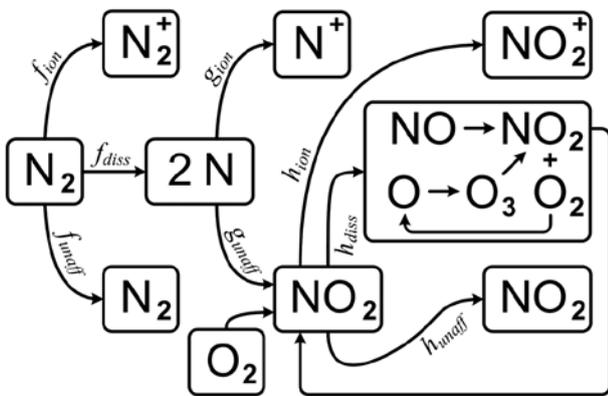


Figure 8: The chemical process of nitrogen compounds of the air in the electric field.

As in the previous case, the yields of the outgoing nitrogen gases along the shortest path between the cathode and the anode are shown in Fig. 9 in the moment of the throughout ionization of O_3 molecules formation – see Fig. 10, which is followed by the electric breakdown in the air gap.

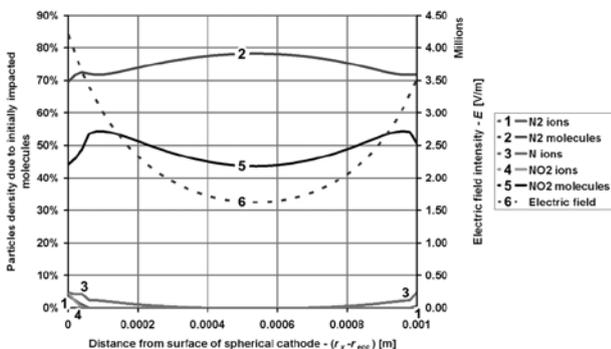


Figure 9: The yields of the chemical process of nitrogen compounds of the air in the electric field along the shortest path between the electrodes.

The ratios between the nitrogen compound particles densities, which are the output of the chemical process, and the initially impacted nitrogen molecules den-

sity, which is the input of the chemical process, are the yields of the chemical process output, indexed as in Fig. 9, are as follows: $y_1(\text{N}_2 \rightarrow \text{N}_2^+)$, $y_2(\text{N}_2 \rightarrow \text{N}_2)$, $y_3(\text{N}_2 \rightarrow \text{N}^+)$, $y_4(\text{N}_2 \rightarrow \text{NO}_2^+)$ and $y_5(\text{N}_2 \rightarrow \text{NO}_2)$.

Because the discharging arc formation depends on the throughout ionization formation, during whose process the ionized ranges of each constituent gas is being formed as a part of its kinetics, the whole mixture kinetics is influenced by each gas in the mixture, although the chemical process of the particular constituent does not depend on the other gases in the mixture. The chemical process of the O_2 - O - O_3 gas mixture in the electric field, and so far its chemism of the throughout ionization formation in this mixture, is unchanging whether it involves pure oxygen or the air. Nevertheless, the kinetics of the throughout ionization formation, and hence of the discharging arc ignition, is influenced by the particles of the other constituents of the air through its physics. Namely, the other constituents particles, as nitrogen compounds particles, are intercepting the electrons, and so reducing the electron collisions to oxygen particles, and beside that they are forming their own ionized ranges, which contribute their part to the ionized ranges growth, and so far to the throughout ionization formation. The N_2 - N - NO_2 gas mixture is also influenced by the other air constituents through its physics, hence the kinetics of the throughout ionization formation is based on an equilibrium between all air constituents in every stage of the ionized ranges growth. The kinetics, weather dealt separately as the O_2 - O - O_3 mixture and the N_2 - N - NO_2 mixture or together as the air, is qualitatively the same, but it is quantitatively different in the transfer functions, pondered by mixing ratio of oxygen (cca 20%) and nitrogen (cca 80%) in the air as modelled through physics, and hence in yield functions of chemism, and further on in concentrations and in the ionized ranges growth until the one of the constituent gases is throughout ionized – Fig. 10.

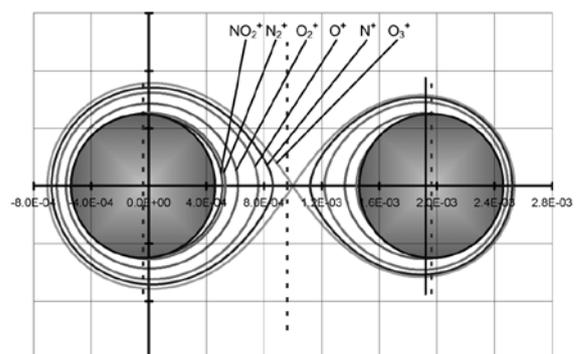


Figure 10: The ionization border lines C_{eqex_ion} as the axial cross-section of the ionization border surfaces for the main constituents of the air just when the throughout ionized path is formed by ozone ions.

In this figure, there are the ionization border lines C_{eqex_ion} representing the ionization border surfaces, as the geometric bodies of the electrodes are rotary symmetrical, for each kind of the constituent ions in the air, where the throughout ionization is achieved by ozone ions. These lines are not the Cassini ovals. The ionization border lines (surfaces) C_{eqex_ion} defined by Eq. (24), circumscribe the ionized range, which consist of the highly ionized and the partly ionized range in Fig. (4), and is complementary to the non-ionized ranges in Eq. (24):

$$R_{ion} = \bar{R}_{non_ion} = \{ T_i(r, \varphi) \leftarrow W_{exm_mx}(r, \varphi, t, p, T, W_{ion}, W_{diss}, m_m) \geq W_{ion} \}$$

for each kind of constituent ions.

If the ionized range of Eq. (27) or the union of all ionized ranges, since the air is gas mixture, includes at least one subset of points that is continuous between any point on the cathode and any point on the anode, the throughout ionization is established. Hereby, any subset of the ionized range or of their union that is continuous between any point on the cathode and any point on the anode is the throughout ionized path, and the discharging arc is formed by the electric breakdown along one of the throughout ionized paths. A voltage over the cathode-anode gap, at which the throughout ionization is established, is the throughout ionization voltage, and a relevant current is the throughout ionization current. If there is no throughout ionized path, the non-ionized gap exists with its distance between the relevant ionized ranges around the cathode and the anode, which is the infimum of the distances between any two of their respective points. The non-ionized gap can be stated due to the whole gas mixture (i.e. air) or separately due to the particular constituent gas – Tab. 2.

Table 2: The characteristic parameters at the throughout ionization of the air gap.

	unit	Σ air	O ₂ ⁺	O ⁺	O ₃ ⁺	O ₃	N ₂ ⁺	N ⁺	NO ₂ ⁺	NO ₂
throughout ionization voltage	kV	2.23	←	←	←	←	←	←	←	←
throughout ionization current	μA	26.6	←	←	←	←	←	←	←	←
non-ionized gap	mm	0	0.76	0.48	0	-	0.92	0.26	0.98	-
ratio of impacted particles	%	-	0.23	0.54	0.41	9.9	0.12	0.79	0.10	39
concentration in air	×10 ⁻¹⁸	-	0.40	0.93	0.71	17	0.14	0.93	0.12	45

The gas medium in the gap, if not noble gas, becomes chemically active under the sufficient electric field intensity in this gap, mostly producing new gaseous

compounds that are more vulnerable to ionization, and hence the mixture throughout ionization voltage is lowering, as well as its breakdown voltage. If the gas medium is the air, the electric field intensity can cause its pollution, since the ozone and the nitrogen dioxide are forming. Certainly, every formation of these two compounds does not mean the real pollution, and so far, in this particular case air quality indices [10] are zero and the category of the air is good.

4. Conclusions

Although the paper deals with the discharging arc ignition and especially the phenomena in early stage of the ionized path building up to the throughout ionized path establishment quite scholastically, it gives some understanding of physical and chemical processes carried on in the narrow gap of the gas mixture in the presence of the non-homogeneous electric field due to the primary and the secondary electron emission, and further on, due to the chemical changes in the gas mixture structure. So far, the arc avoiding and the arc extinguishing methods are clarified on the basis of the mathematical models of the drawn arc formation [1], and of the discharging arc formation model. On the basis of the discharging arc formation model some scientific conclusions and quantitatively evaluated results are obtained as stated in the following highlights:

- the kinetics of the processes in the gas medium between two electrodes shows that the discharging arcs in their ignition stage are not significantly determined by the contact materials and their arc forming affinity [1], except for the secondary emission, which hastens the throughout ionized path formation and hence discharging arc ignition,
- the discharging arc ignition strongly depends on physical and chemical properties and reactions of gas mixture medium, and especially chemical reactions increase the ionized path growth;
- so far, the methods to avoid discharging arcs are not based on the proper choice of contact materials with the low affinity to form arcs, but the avoiding and extinguishing is performed by specially designed arc quenching electric circuits, minimizing the transient voltages over the gap of electric contacts during their manoeuvres;
- the discharging arcs are diminished by applying parallel capacitance to inductive loads or to the electric contacts in d.c. electric circuits to minimize the displacement current and the peak value of the overvoltage;
- the discharging arcs are avoided by using inert gas medium and/or low gas pressure and even vacuum in sealed electric contacts;

- the discharging arc are extinguished by lengthening the throughout ionized path for instance by the application of magnetic field perpendicular to the electric field lines in the contact gap, and hence producing force on the flowing electrons;
- further on, in cases of some severe electric fields, as in the surroundings of EHV (extra high voltage) power lines, the discharge arc formation model, applied to appropriate geometrical and electrical parameters associated with the electrodes and the environment, gives the quantitative estimation method of ozone and nitrogen dioxide pollution in the air.

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