Performance Analysis of a Memristor - Based Biquad Filter Using a Dynamic Model

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Abstract: New kinds of programmable amplifiers, adaptive filters, and programmable oscillators can be designed using a new fundamental circuit element memristor. Application of the memristor to analog filters can result in many new properties especially thanks to its variable memristance. In this paper, by using linear drift memristance model, a memristor-based biquad analog filter is examined. The filter is simulated using its dynamic model. The linear dopant drift model of TiO₂ memristor is used in the simulations. Simulations have shown that the filter components such as gain and quality factor can be adjusted using the memristor in biquad filter. It has also been observed that memristor may go into saturation at very low frequencies and very low charge values. Considering the results of this study, the tunable memristance gives filters an adjustable characteristic which cannot be obtained by traditional resistors. Results presented in this study can also be considered when designing biquad filters with memristors to ensure their stability and high performance.

Keywords: Memristor, analog filters, memristor-based filter design, biquad filters

1 Introduction

Memristor was declared as the missing circuit element in 1971 [1]. It can be thought as a nonlinear resistor whose resistance or memristance depends on electrical charge which have passed through it. Almost 40 years later than its prediction, a memristive system which behaves as a memristor has been announced as fabricated [2]. This solid state realization has resulted in a large-scale interest in memristor. However, no commercially and practically available memristor exists yet. Memristors can be used in analog circuits and may provide many new additional features. Various

Analiza učinkovitosti bikvadrantnega filtra na osnovi memristorja z uporabo dinamičnega modela


Ključne besede: Memristor, analogni filtri, memristor filtri, bikvadrantni filter

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SPICE macro-models, memristor emulators exhibiting memristor-like behavior and some analog and chaotic applications have already been presented in literature [3-10].

It is thought that application of the memristor to analog filters can result in many new properties especially because of its variable memristance. Some studies are already presented in the literature related to application of the memristor to analog filters such as band-pass adaptive filter based on 

\( \text{VO}_2 \) memristor [11], a first-order filter for sensing resistive memory based on a memristor and a capacitor [12], low-pass and high-pass filter design using TaO\(_x\) based memristor [13], first-order and a second-order low-pass filters employing Cu/ZnO/Cu based memristive structures [14], analyzes of first-order low-pass and second-order band-pass memristor-based filters using PSpice based Boundary Condition Model (BCM) [15], a first-order memristor based low-pass filter using its dependent voltage source based PSpice model [16].

Tow-Thomas biquad filter [17-19] is used in many analog signal processing applications. Current or voltage mode Tow-Thomas filters have been introduced and their quality factor and cut-off frequency are electronically tunable by adjusting some circuit components or the biasing current of CMOS active blocks etc [20-24]. In this study, a memristor-based Tow-Thomas Biquad filter is designed, it uses a resistance tuning circuit and does not need any active blocks to adjust the filter parameters. It has also an advantage: the memristor is a nonvolatile memory and, if the power source is turned off and back again, the memristor remembers the charge which passed through it and the filter starts operating with the last operating points set.

In this study, the feedback resistor of the first stage in the Tow-Thomas filter is replaced with a memristor and the memristor-based filter properties are inspected using simulations. Linear dopant drift model of TiO\(_2\) memristor [2,3] and it is also used in the simulations performed. For the first time in the literature to the best of our knowledge, we will demonstrate the effect of polarity of memristor on waveforms of memristor-based filters. Results of this study can be used for designing memristor-based filters when a memristor as a separate or integrated circuit component element becomes commercially available in the market.

Structure of the paper is as follows. The second section, which follows this introduction, summarizes linear drift modeling of a memristor. In the third section, the memristor based Tow-Thomas biquad filter is introduced. In the fourth section, dynamic model of the memristor-based biquad filter is given. In the fifth section, simulation results performed on Simulink\textsuperscript{TM} toolbox of MATLAB\textsuperscript{TM} are provided. Time domain waveforms of current, voltage, charge and memristance are analyzed. The effect of polarity is analyzed. Gain responses of the band-pass (BP) and low-pass (LP) biquad filters with respect to frequency and as a function of variable memristance are also inspected. Finally, in the last chapter, the performance and constraints of memristor based filters and also its new possibilities on analog filter design are discussed.

## 2 Linear drift TiO\(_2\) memristor model

A memristor is a nonlinear circuit element whose voltage to current ratio or its instantaneous resistance depends on electrical charge which have passed through it [1]. A memristor can also be modeled as flux dependent [1]. In this study, charge-controlled memristor model is used. The instantaneous resistance of a memristor is called memristance and memristance function of a charge-controlled memristor is given as

\[
M(q) = \frac{d\varphi(q)}{dq}
\]  

where \( q \) is the memristor charge and \( \varphi \) is the memristor flux, which are equal to integration of current with respect to time and integration of voltage with respect to time, respectively. They are calculated as

\[
q(t) = \int_{-\infty}^{t} i(\tau) d\tau
\]

\[
\varphi(t) = \int_{-\infty}^{t} v(\tau) d\tau
\]

In this work, the charge dependent model of TiO\(_2\) memristor with linear dopant drift speed as given in [2] is used. Memristance function of TiO\(_2\) memristor with linear dopant drift system is given as

\[
M(q) = \frac{d\varphi}{dq} = R_{\text{OFF}} \left( 1 - \frac{H_i R_{\text{ON}}}{D^2} q(t) \right)
\]

Memristance function can also be simplified as

\[
M(q) = M_0 - K_q q(t)
\]

where \( M_0 \) is the maximum memristance of the memristor, and \( M_0 = R_{\text{OFF}} \). \( K_q \) is the charge coefficient of the memristor and \( q(t) \) is the instantaneous memristor charge.

The minimum memristor memristance is given as

\[
M_{\text{sat}} = M_0 - K_q q_{\text{sat}}
\]
where \( q_{\text{SAT}} \) is the maximum memristor charge. The terminal equation of a charge-controlled memristor is expressed as

\[
v = M(q)i
\]

(7)

3 Biquad filter

In this section, a Tow-Thomas biquad filter is first briefly explained and then the memristor-based biquad filter is introduced.

3.1 Tow-thomas biquad filter

A two-integrator loop biquad filter topology where all three op-amps are used in a single-ended configuration is shown in Figure 1 [17-19]. It is called as a Tow-Thomas biquad after its inventors. It has two integrator circuits and an inverting amplifier at the last stage. This configuration has a BP output, a LP output and one more LP output in reversed phase which are taken from nodes \( V_{\text{BP}} \), \( V_{\text{LP}} \) and \( V_{\text{LP}^-} \) respectively. In this topology high-pass output is no longer available as it is the result of all op-amps being in the single-ended mode [20,21].

\[ H_{\text{BP}}(s) = \frac{V_{\text{BP}}}{V_{\text{IN}}} = -\frac{s}{s^2 + \frac{\alpha_b}{Q} + \alpha_b^2} \]

(8)

The transfer function for the BP filter given in Figure 1:

\[ H_{\text{BP}}(s) = \frac{V_{\text{BP}}}{V_{\text{IN}}} = -\frac{s}{s^2 + \frac{1}{C_1R_1} + \frac{1}{C_1C_2R_2R_6}} \]

(9)

Therefore, some important parameters of the filter in Figure 1 can be expressed as follows.

The BP gain at the cut-off or resonant frequency:

\[ H_{\text{BP}}(s) = \frac{R_1}{R_1} \]

(10)

The cut-off frequency:

\[ \omega_b = \sqrt{\frac{C_1C_2R_2R_6}{R_4}} \]

(11)

The filter quality factor:

\[ Q = \frac{C_1C_2R_2^2}{C_1R_2R_6} \]

(12)

For the simplicity, by taking \( R_4=R_6 \) the filter parameters become

\[ H_{\text{BP}}(s) = \frac{V_{\text{BP}}}{V_{\text{IN}}} = -\frac{s}{s^2 + \frac{1}{C_1R_1} + \frac{1}{C_1C_2R_2R_6}} \]

(13)

The transfer function for the LP filter given in Figure 1 is:

\[ H_{\text{LP}}(s) = \frac{V_{\text{LP}}}{V_{\text{IN}}} = -\frac{R_1}{s^2 + \frac{1}{C_1R_1} + \frac{1}{C_1C_2R_2R_6}} \]

(17)

By taking \( R_4=R_6 \):

\[ H_{\text{LP}}(s) = \frac{V_{\text{LP}}}{V_{\text{IN}}} = -\frac{1}{s^2 + \frac{1}{C_1R_1} + \frac{1}{C_1C_2R_2R_6}} \]

(18)

Its gain at LP frequencies is given as

\[ H_{\text{LP}}(s) = \frac{R_1}{R_1} \]

(19)

Both types of filter transfer functions have the same denominator. Thus they have the same cut-off frequencies and quality factors, which are given in and .
3.2 Memristor-based biquad filter

The memristor based Tow-Thomas biquad filter is obtained by replacing resistor $R_2$ with a memristor designated as $M(q)$. Considering its polarity, a memristor can be placed into the filter in two ways as shown in Figure 2 and Figure 3.

In order to adjust the quality factor of the filters without affecting the cut-off frequency, a memristor will be used instead of resistor $R_2$. By replacing $R_1$ with the memristor, $M(q)$, the gain of the band-pass filter at the resonant frequency also becomes adjustable with memristance. For the simplicity, by taking that $R_4=R_5$ and assuming that the memristor has almost constant memristance, i.e. a small memristance change, the resonant frequency gain of the BP and the quality factor of the both BP and LP memristor-based biquad filters can be expressed as follows:

\[
H_{BP}(q) = \frac{M(q)}{R_1} \tag{20}
\]

\[
Q = \sqrt{\frac{C_1(M(q))^2}{C_2R_1R_6}} \tag{21}
\]

**Figure 2:** Tow-Thomas biquad filter with positive polarized memristor

**Figure 3:** Tow-Thomas biquad filter with negative polarized memristor

Since some of the filter parameters can be adjusted with the memristor memristance, a tuning circuit is needed for this purpose. In Figure 4 (a) memristance value set circuit is shown and its combination with Tow-Thomas biquad filter with positive polarized memristor is given in Figure 4 (b).

S$_1$ and S$_4$ are turned on to decrease memristance. S$_2$ and S$_3$ are turned on to increase memristance. For a simplified filter operation, it is preferred to make initial capacitor charge equal to zero. S$_1$ and S$_3$ can both be turned on to short-circuit the capacitor C$_1$ for this purpose during memristance tuning.

4 Dynamic system model of memristor based biquad filter

Assuming there is no saturation in the circuit, State-space representation of the memristor based Tow-Thomas biquad filter is given as

\[
\frac{dv_{c_1}}{dt} = \frac{1}{C_1} \left( -\frac{v_{c_1}}{M(q)} + \frac{R_5}{R_4R_6} v_{c_2} + \frac{1}{R_4} v_{in} \right) \tag{22}
\]

\[
\frac{dv_{c_2}}{dt} = -\frac{v_{c_2}}{R_4C_2} \tag{23}
\]

The filter outputs are

\[
v_{BP} = -v_{c_1} \tag{24}
\]

\[
v_{LP} = -v_{c_2} \tag{25}
\]

\[
v_{LP} = \frac{R_6}{R_4} v_{c_2} \tag{26}
\]
When the memristor is added, due to its memristance, which is a nonlinear function, an analytical solution for the memristor-based biquad filter circuit output voltage in time or frequency domain cannot be obtained. That’s why its dynamic model was designed and simulated with Simulink™ toolbox of MATLAB™. Simulink is commonly used to model, analyze and simulate dynamic systems and it has comprehensive block library which can be used to simulate linear, non-linear or discrete systems.

The block diagram of dynamic system and sub-block of linear drift memristor are shown in Figure 5 and Figure 6, respectively. Block diagram of the filter is constructed based on the dynamic model of the filter using a clock signal, a sinusoidal source, a constant, gains, integrators, mathematical operation blocks, and sinks from Simulink library. One of the integrators used is a saturable integrator used in the memristor sub-block diagram to limit memristor memristance between $M_{\text{SAT}}$ and $M_0$.

5 Simulation results

A sinusoidal input voltage of $v(t)=V_m \times \sin(\omega t)$ is applied to the input of the memristor-based biquad filter in all cases. Unless otherwise noted, the following circuit parameters are used in simulations: $V_m=0.5V$, $R_1=5k\Omega$, $R_2=5k\Omega$, $R_3=5k\Omega$, $R_4=5k\Omega$, $R_5=5k\Omega$, $C_1=300nF$ and $C_2=300nF$. And memristor parameters are taken as $M_0=40k\Omega$, $M_{\text{SAT}}=100\Omega$.

Current-voltage waveforms, memristor hysteresis loop, memristance and memristor charge characteristics are illustrated in time domain. Also the effect of memristor polarity is analyzed. Filter output voltages are simulated in time domain. Filter gain responses are obtained with respect to frequency taking $M_{\text{avg}}$ or $M(q_0)$ as parameter where $q_0$ is the initial or the average charge of the memristor.

5.1 Memristor characteristics and the effect of polarity on filter waveforms

Memristor current-voltage, hysteresis loop, memristance-memristor charge waveforms are presented in Figure 7, Figure 10, Figure 13 for 2 Hz; Figure 8, Figure 11, Figure 14 for 100 Hz and Figure 9, Figure 12, Figure 15 for 1 kHz, respectively.

As it can be seen from Figure 7, Figure 10 and Figure 13 there is distortion in memristor waveforms at very low frequencies and low $q_{\text{SAT}}$ values. Saturation of memristor, occurring at very low frequencies, is examined by taking $q_{\text{SAT}}=0.075 \mu C$. If the frequency is made somewhat higher or a memristor with a sufficiently higher $q_{\text{SAT}}$ value is chosen, this distortion either is reduced or disappears.

The saturation occurs at low frequencies since the period is high enough to push current to saturate the memristor i.e. to make its charge equal to $q_{\text{SAT}}$ for some interval(s) as seen in Figure 7, Figure 10 and Figure 13. During saturation, all the waveforms, the memristor current, voltage, charge and memristance, become distorted as seen in Figure 7, Figure 10 and Figure 13. The saturation disappears since at high frequencies a memristor behaves as a resistor or the period is not high enough to push current to saturate the memristor, i.e. is unable to make its charge equal to $q_{\text{SAT}}$ for some interval(s) as seen in Figure 8, Figure 9, Figure 11, Figure 12, Figure 14 and Figure 15.
Figure 7: Memristor voltage and current with respect to time for $q_{\text{SAT}}=0.075\mu\text{C}$, $q_0=0.1 \times q_{\text{SAT}}$ at $f=2\text{Hz}$, with (a) positive polarity (b) negative polarity

Figure 8: Memristor voltage and current with respect to time for $q_{\text{SAT}}=2.5\mu\text{C}$, $q_0=0.1 \times q_{\text{SAT}}$ at $f=100\text{Hz}$, with (a) positive polarity (b) negative polarity

Figure 9: Memristor voltage and current with respect to time for $q_{\text{SAT}}=2.5\mu\text{C}$, $q_0=0.1 \times q_{\text{SAT}}$ at $f=1\text{kHz}$, with (a) positive polarity (b) negative polarity

Figure 10: Memristor I-V characteristics for $q_{\text{SAT}}=0.075\mu\text{C}$, $q_0=0.1 \times q_{\text{SAT}}$ at $f=2\text{Hz}$, with (a) positive polarity (b) negative polarity
Figure 11: Memristor I-V characteristics for $q_{\text{SAT}}=2.5$µC, $q_0=0.1\times q_{\text{SAT}}$ at $f=100$Hz, with (a) positive polarity (b) negative polarity

Figure 12: Memristor I-V characteristics for $q_{\text{SAT}}=2.5$µC, $q_0=0.1\times q_{\text{SAT}}$ at $f=1$kHz, with (a) positive polarity (b) negative polarity

Figure 13: Memristance and memristor charge with respect to time for $q_{\text{SAT}}=0.075$µC, $q_0=0.1\times q_{\text{SAT}}$ at $f=2$Hz, with (a) positive polarity (b) negative polarity

Figure 14: Memristance and memristor charge with respect to time for $q_{\text{SAT}}=2.5$µC, $q_0=0.1\times q_{\text{SAT}}$ at $f=100$Hz, with (a) positive polarity (b) negative polarity
Figure 15: Memristance and memristor charge with respect to time for \( q_{\text{sat}} = 2.5 \mu\text{C}, \ q_0 = 0.1 \times q_{\text{sat}} \) at \( f = 1\text{kHz} \), with (a) positive polarity (b) negative polarity.

The effects of the memristor polarity on the memristor waveforms are also analyzed and the corresponding curves are presented in Figure 7 - Figure 15 (b). If polarity of the memristor is switched as shown in Figure 3, the memristor currents for the reversed polarity are obtained by first 180° phase-shifting of the memristor current for the positive polarity and then flipping it with respect to horizontal-axis as shown in Figure 7 and Figure 8. At 1 kHz, since the frequency is sufficiently high, the memristor behaves as if it was a resistor, the polarity has no effect and the memristor voltages and currents for both polarities can be regarded as identical and sinusoidal as seen in Figure 9. For the reversed polarity, the memristor memristance and charge can be obtained by either phase-shifting of the memristor memristance and charge for the positive polarity by 180° or flipping them with respect to vertical axis as shown in Figure 13 and Figure 14. However, at 1 kHz, since the frequency is sufficiently high, the memristor behaves as a resistor, the polarity has no effect and the memristor memristance and charge can be regarded as constants as seen in Figure 15.

5.2 Filter output voltages with respect to time

Output voltages of memristor-based LP and BP filters are simulated. Corresponding time domain waveforms are provided in Figure 16 and Figure 17. As shown in Figure 16 and Figure 17, if the memristor has a low \( q_{\text{sat}} \) value (\( q_{\text{sat}} = 0.075 \mu\text{C here} \)), it becomes saturated at very low frequencies as demonstrated in previous section and the output voltage has some distortion. The filter with either higher \( q_{\text{sat}} \) value (\( q_{\text{sat}} = 2.5 \mu\text{C here} \)) or with the increasing of the frequency hasn’t any distortion since it does not get saturated. That’s why the memristor \( q_{\text{sat}} \) value should be chosen high enough not to saturate the memristor over all the operating frequency range for a well-designed filter.

Figure 16: LP biquad filter input and output voltages as a function of time for \( q_0 = 0.1 \times q_{\text{sat}} \) with positive polarity at \( f = 2\text{Hz}, \ q_{\text{sat}} = 0.075\mu\text{C (b) f=100Hz, q}_{\text{sat}} = 2.5\mu\text{C} \)

5.3 Gain characteristics of filters

Gain characteristics of the biquad LP and BP filters with respect to frequency by taking \( M_{\text{avg}} \), which is the average memristor memristance in steady-state, as parameter are shown in Figure 18. As it can be seen from the Figure 18 (a), (b) and (c), adjustable memristance in both BP and LP filters provides variable quality factor. Also an adjustable gain in BP biquad is obtained by inconstancy of the memristance as shown in Figure 18 (c).
6 Conclusions

In this paper, a memristor-based biquad analog filter is examined by using a linear drift model of a memristor. Since an analytical solution was not available, the characteristics of the filter are observed by means of simulations.

Simulations have shown that the quality factor and BP filter gain at the resonant frequency can be adjusted by varying the average memristance of a memristor in a biquad filter. The effect of the memristor polarity on the biquad filter is also inspected. It has been found that, at low frequencies, the polarity also defines the shape of the memristor current, voltage and memristance waveforms but, at high frequencies, it has no effect, i.e. all waveforms have the same shape for both polarities since the memristor behaves as a resistor.

It has also been observed that the memristor, depending on its $q_{\text{sat}}$ value, may go into saturation at very low frequencies. Above the very low operation frequencies the biquad filter operates without saturation but with some distortion due to varying memristance. At high frequencies, the memristor behaves similar to a resistor and the filter operates well without distortion.

The undesired saturation or a big memristance perturbation in biquad filters results in distortion at output voltage waveforms. These issues for memristor-based biquad filters are considered for the first time in literature. That’s why special precautions should be taken when designing memristor-based biquad analog filters regarding the input signal magnitudes and operation frequencies. The analysis of the filter can also be done using more complex models when they become available.
It is expected that the new circuit element, memristor, can contribute new properties to analog circuits. According to the results presented in this study, the variable memristance gives the memristor-based biquad filter adjustable characteristics, which cannot be mimicked with traditional resistors. The memristor-based biquad filter is able to tune the filter parameters, the gain at the resonant frequency, and the BP filter quality factor, i.e. the bandwidth. Results of this study can be used to design memristor-based biquad filters, fulfilling their performance and stability requirements.

7 References